九十四學年第二學期 PHYS2320 電磁學 期末考試題(共一頁)

[Griffiths Ch. 9, 10, 12]　2006/06/13,　10:10am–12:00am, 教師：張存續

記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

1. Explain the following items as clear as possible.
   (a) Invariant quantity and conserved quantity (7%)
   (b) Lienard-Wiechert potentials (7%)
   (c) Phase velocity and group velocity (7%)
   (d) Lorentz gauge and Coulomb gauge (7%)
   (e) Einstein's postulates for the special relativity (6%)
   (f) Hidden momentum (6%)

2. (a) $\mathbf{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$, $\mathscr{r} = |\mathbf{r} - \mathbf{r}'| = c(t - t_r)$, and $\mathbf{v} = \mathbf{v}(t_r)$, find $\nabla \mathscr{r}$ and $(\mathbf{\mathscr{r}} \cdot \nabla)\mathbf{v}$. (10%)
(Note: Express your answer in terms of ∇t_r.)
(b) The transformations between two inertial systems S and $\bar{S}$ are $\bar{x} = \gamma(x - vt)$ and $\bar{t} = \gamma(t - vx/c^2)$. Show that when $\Delta t = 0$, $\Delta x = \Delta\bar{x}/\gamma$; but when $\Delta\bar{t} = 0$, $\Delta\bar{x} = \Delta x/\gamma$. Explain why the length relations depend on simultaneity. (10%)
(Hint: Specify the proper length, $\Delta\bar{x}$ or Δx, in each case.)

3. A coaxial transmission line consists of a long straight wire of radius a and a cylindrical conducting sheath of radius b. The electric and magnetic fields are given by: $\mathbf{E}(s,\phi,z,t) = \dfrac{E_0\cos(kz - \omega t)}{s}\hat{\mathbf{s}}$ and $\mathbf{B}(s,\phi,z,t) = \dfrac{E_0\cos(kz - \omega t)}{cs}\hat{\boldsymbol{\phi}}$,
(a) Write Maxwell's equations in free space (no source) and show that **E** and **B** obey Gauss's law and Faraday's law. (10%)
(b) Find the charge density, $\lambda(z,t)$, and the current, $I(z,t)$, on the inner conductor. (10%)
(Note: Use the cylindrical coordinate: $\nabla\cdot\mathbf{v} = \dfrac{1}{s}\dfrac{\partial(sv_s)}{\partial s} + \dfrac{1}{s}\dfrac{\partial v_\phi}{\partial \phi} + \dfrac{\partial v_z}{\partial z}$
and $\nabla\times\mathbf{v} = (\dfrac{1}{s}\dfrac{\partial v_z}{\partial\phi} - \dfrac{\partial v_\phi}{\partial z})\hat{\mathbf{s}} + (\dfrac{\partial v_s}{\partial z} - \dfrac{\partial v_z}{\partial s})\hat{\boldsymbol{\phi}} + \dfrac{1}{s}[\dfrac{\partial}{\partial s}(sv_\phi) - \dfrac{\partial v_s}{\partial\phi}]\hat{\mathbf{z}}$.

4. The intensity of sunlight hitting the Earth is about 1300 W/m^2.
(a) Find the amplitudes of the electric field (in V/m) and magnetic field (in T). (10%)
   (Note: $c = 3\times10^8$ m/s and $\varepsilon_0 = 8.85\times10^{-12}$ C^2/N$\cdot$m^2)
(b) If sunlight strikes normally a perfect reflector (say, a mirror) of area 10 m^2, what force does it exert? (10%)

1.

(a) Invariant quantity: same value in all inertial frames.

Conserved quantity: same value before and after some process.

(b) Lienard-Wiechert potentials are the scalar and vector potentials for a moving point charge.

$V(\mathbf{r},t) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{(\mathfrak{r} - \boldsymbol{\mathfrak{r}} \cdot \mathbf{v}/c)}; \mathbf{A}(\mathbf{r},t) = \dfrac{\mathbf{v}}{c^2} V(\mathbf{r},t)$.

(c) Phase velocity $v_p = \dfrac{\omega_0}{k_0}$; group velocity $v_g = \dfrac{d\omega}{dk}$

(d) The Lorentz gauge $\nabla \cdot \mathbf{A} + \mu_0 \varepsilon_0 \dfrac{\partial V}{\partial t} = 0$: It treat V and $\mathbf{A}$ on an equal footing and is particularly nice in the context of special relativity.

The coulomb gauge $\nabla \cdot \mathbf{A} = 0$: The scalar potential is particularly simple to calculate, but the vector potential is very difficult. It is suitable for the static case.

(e) Einstein's postulates for the special relativity

1. The principle of relativity: All physical laws have the same form in all inertial frames.
2. The universal speed of light: The speed of light in free space is the same in all inertial frames. It does not depend on the motion of the source or the observer.

(f) Hidden momentum is strictly relativistic, and purely mechanical.

2. (a) $\nabla \mathfrak{r} = \nabla c(t - t_r) = -c \nabla t_r$

$$(\boldsymbol{\mathfrak{r}} \cdot \nabla)\mathbf{v} = (\mathfrak{r}_x \frac{\partial}{\partial x} + \mathfrak{r}_y \frac{\partial}{\partial y} + \mathfrak{r}_z \frac{\partial}{\partial z})\mathbf{v}$$

$$= (\mathfrak{r}_x \frac{d\mathbf{v}}{dt_r} \frac{\partial t_r}{\partial x} + \mathfrak{r}_y \frac{d\mathbf{v}}{dt_r} \frac{\partial t_r}{\partial y} + \mathfrak{r}_z \frac{d\mathbf{v}}{dt_r} \frac{\partial t_r}{\partial z}) = \mathbf{a}(\boldsymbol{\mathfrak{r}} \cdot \nabla t_r)$$

(b) $\Delta t = 0 \Rightarrow \Delta\bar{x} = \gamma \Delta x \Rightarrow \Delta x = \dfrac{1}{\gamma} \Delta\bar{x}$. In this case $\Delta\bar{x}$ is the proper length.

$$\Delta\bar{t} = 0 \rightarrow \Delta t = \frac{v}{c^2} \Delta x$$

$$\Delta\bar{x} = \gamma(\Delta x - v\Delta t) = \gamma(\Delta x - \frac{v^2}{c^2}\Delta x) = \gamma \frac{1}{\gamma^2} \Delta x = \frac{1}{\gamma} \Delta x$$

$\Delta\bar{x} = \dfrac{1}{\gamma} \Delta x$ (Δx is the proper length)

Two events that are simultaneous in one inertial system are not simultaneous in another.

All physical laws have the same form in all inertial frames.

The moving objects are shortened.

3. (a) $\nabla\cdot\mathbf{E}=0; \nabla\cdot\mathbf{B}=0; \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}; \nabla\times\mathbf{B}=\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$

$$\mathbf{E}(s,\phi,z,t)=\frac{E_0\cos(kz-\omega t)}{s}\hat{\mathbf{s}} \text{ and } \mathbf{B}(s,\phi,z,t)=\frac{E_0\cos(kz-\omega t)}{cs}\hat{\boldsymbol{\varphi}}$$

$$\nabla\cdot\mathbf{E}=\frac{1}{s}\frac{\partial}{\partial s}(sE_s)+\frac{1}{s}\frac{\partial E_\phi}{\partial\phi}+\frac{\partial E_z}{\partial z}=\frac{1}{s}\frac{\partial}{\partial s}(E_0\cos(kz-\omega t))=0; \quad\Rightarrow\ \nabla\cdot\mathbf{E}=0$$

$$\nabla\times\mathbf{E}=(\frac{1}{s}\frac{\partial E_z}{\partial\phi}-\frac{\partial E_\phi}{\partial z})\hat{\mathbf{s}}+(\frac{\partial E_s}{\partial z}-\frac{\partial E_z}{\partial s})\hat{\boldsymbol{\varphi}}+\frac{1}{s}[\frac{\partial}{\partial s}(sE_\phi)-\frac{\partial E_s}{\partial\phi}]\hat{\mathbf{z}}$$

$$=\frac{\partial E_s}{\partial z}\hat{\boldsymbol{\varphi}}-\frac{1}{s}\frac{\partial E_s}{\partial\phi}\hat{\mathbf{z}}=-k\frac{E_0\sin(kz-\omega t)}{s}\hat{\boldsymbol{\varphi}}$$

$$-\frac{\partial\mathbf{B}}{\partial t}=-\omega\frac{E_0\sin(kz-\omega t)}{cs}\hat{\boldsymbol{\varphi}}=-k\frac{E_0\sin(kz-\omega t)}{s}\hat{\boldsymbol{\varphi}} \quad\Rightarrow\ \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

(b)

To determine λ, use Gauss's law for a cylinder of radius a and length dz.

$$\oint_{s=a}\mathbf{E}\cdot d\mathbf{a}=2\pi a\frac{E_0\cos(kz-\omega t)}{a}dz=\frac{Q_{\text{enc}}}{\varepsilon_0}=\frac{\lambda}{\varepsilon_0}dz \quad\Rightarrow\ \lambda=\varepsilon_0 2\pi E_0\cos(kz-\omega t)$$

To determine I, use Ampere's law for a circle of radius a.

$$\oint_{s=a}\mathbf{B}\cdot d\mathbf{l}=2\pi a\frac{E_0\cos(kz-\omega t)}{ca}=\mu_0 I \quad\Rightarrow\ I=\frac{2\pi E_0}{\mu_0 c}\cos(kz-\omega t)$$

4. (a)

$$I\equiv\langle\mathbf{S}\rangle=\frac{1}{2}c\varepsilon_0E_0^2=1300\text{ W/m}^2$$

$$E_0=\sqrt{\frac{2\times1300}{c\varepsilon_0}}=\sqrt{\frac{2\times1300}{3\times10^8\times8.85\times10^{-12}}}=990\ \text{V/m}$$

$$B_0=E_0/c=3.3\times10^{-6}\ \text{T}$$

(b) $$F=\overbrace{P}^{\text{pressure}}\cdot\overbrace{A}^{\text{area}}=\frac{\overbrace{\Delta p}^{\text{momentum}}}{\Delta t}=\frac{2I\Delta t}{c\Delta t}\cdot A=8.6\times10^{-5}\ \text{N}$$